Review Set 1

This Review Set asks you to prepare answers to questions on regular languages and finite automata. Each of the questions has a short answer. You may discuss the Review Set with other students and work on the problems together.

1. Consider the following languages over the alphabet \( \Sigma = \{a, b\} \).
   
   - \( L_1 \): All strings that contain at least three \( a \)'s.
   - \( L_2 \): All strings that contain at most one \( b \).
   - \( L_3 \): All strings that contain at least three \( a \)'s but at most one \( b \).
   - \( L_4 \): All strings that contain no \( b \)'s.

   For each of the languages \( L_1, L_2, L_3 \) and \( L_4 \), give a deterministic finite automaton (DFA). (You should thus give four separate DFAs.)

   Aside: This example illustrates that regular languages are closed under intersection. Note that \( L_3 = L_1 \cap L_2 \).

2. Consider the following DFA over the alphabet \( \Sigma = \{a, b\} \).

   
   ![DFA Diagram]

   Give a one-sentence description of the language recognized by the DFA. Write a regular expression for the same language.

3. Consider the following languages:

   - \( L_1 \) is all strings over the alphabet \( \Sigma = \{x, y\} \) where either \( x \) occurs an odd number of times or \( y \) occurs an odd number of times (or both).
   - \( L_2 \) is all strings over the alphabet \( \Sigma = \{x, y, z\} \) where either \( x \) occurs an odd number of times or \( y \) occurs an odd number of times or \( z \) occurs an odd number of times (or both, or all three).
Give a non-deterministic finite automaton (NFA) for the the languages \( L_1 \). Then give a separate NFA for \( L_2 \).

**Aside:** Non-deterministic finite automata are no more powerful than DFAs in terms of the languages they can describe. They can be exponentially more succinct than DFAs, however.

4. Determine whether or not the following languages are regular. Explain why in one or two sentences.

- \( L_1 \) is all strings over the alphabet \{ (, ) \} where the parentheses are balanced. For example, \( (()()) \in L_1 \) but \( () \notin L_1 \).
- \( L_2 \) is all unique words that are printed in *Programming Language Pragmatics* by Michael L. Scott.
- \( L_3 \) is all 10-digit numbers that are prime.
- \( L_4 \) is the Ocaml language (as described in its reference manual). The alphabet is the set of all tokens and the language is the set of all valid Ocaml programs. \( L_4 \) is not regular; give two reasons why. **Aside:** This explains why we cannot use a lexer to parse languages like Cool or Ruby or C.

5. Give one advantage and one disadvantage of system described in Backus’ *Speedcoding* paper.